Numerical Large Deflection Analysis of Annular Plates

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Abstract

Plates are widely used structures with one size being much smaller than two others. It is known that the loads on the plates are perpendicular to the plate surface. In today's structures, annular plates are designed according to the purpose of use such as storage tanks, machine industry and many engineering areas. In this study, numerical large deflection analyze of annular plates is studied. The solution of the problem is performed based on finite element method. The solutions obtained for different boundary conditions are compared with each other. Simply supported, fixed annular plates carrying uniformly distributed load were investigated by analytical and numerical methods. Stress, strain and displacement expressions are obtained for the annular plates numerically. The results of the numerical study are compatible with the analytical results based on theory of elasticity with large displacement theory of thin plates. It is observed that large deflection analyze of annular plates has a significant place to design engineering structures realistically.

Keywords: *Annular plate, Large deflection, Modeling, Thin plate*

1 Introduction

It is known that one dimension of plates is much smaller the other dimensions. Plates might have fixed, simply supported and free boundary conditions. The property of the plates is that the loads on them are perpendicular to the plate surface. The plates which determined according to the minimum thickness standards can be circular, rectangular or triangular according to the architectural purposes. There are two main options to analyze plates such as analytic or approximate solutions. Analytic solution of plates is important one, it is about the loads and boundary conditions. Approximate methods are composed of energy and numerical methods. Finite Difference Method, Finite Element Method and Boundary Element Method make up numerical methods. Large deflections which become under various loading on plates are well known and there are many researches working on this subject. The obtained results by linearization can be sufficient in many cases of the engineering fields. However, real material's stress-strain relation behaviors are not linear. Taking this fact into consideration, large deflections cannot be analyzed by analytic methods all the times. When this is the case, approximate and numerical methods should be used.

There are many studies on this subject. Some of these are mentioned in this section. Simply supported circular plates under uniform loading are investigated by analytical and numerical methods in their study. The stress, strain and displacement expressions are derived for the plates using the analytical methods based on small displacement theory of thin plates. In addition to these, the numerical solution of the problem based on finite element method is obtained by using SAP 2000 software package. Finally, the results obtained by the analytical methods for different plate thicknesses are compared with each other and with the solutions obtained by the finite element method (Akis and Coran,2014). The finite element method is used to solve the large-deflection bending problem of annular and circular bimodules plates. The material model suggested by Jones is adopted to represent the bilinear stress-strain relations of bimodules material. The resulting nonlinear equations are solved, using the Newton-Raphson iterative procedure. In the case of bimodules composite structures, at any point, the position of neutral axis is a function of the state of stress at that point and is evaluated following iterative procedures. Numerical work has been done for two distinct types of boundary conditions, clamped and simply

supported cases (Srinivasan and Ramachandra, 1988). Geometrically nonlinear bending analysis of clamped circular plates under axisymmetrical transverse load is made in this computational study. The thickness of the plate is uniform, and the plate material is assumed to be isotropic and homogeneous. Since both the plate geometry and the loading are axisymmetric a set of nonlinear ordinary differential equations are solved in the paper. The system of nonlinear algebraic equations which is obtained by the finite difference method is solved by the Newton-Raphson method. The boundary conditions at the support and at the center of the plate are satisfied exactly. The accuracy of the results is verified by checking the maximum deflection with the results available in the literature (Altekin and Yükseler, 2011). The geometrically nonlinear bending analysis of a circular plate with a concentric circular hole. Axisymmetric transverse load is considered in the study. The analysis is made for several boundary conditions including simply supported edge (S), or clamped edge (C), or free edge (F). The thickness of the plate is uniform. The material is assumed to be isotropic and homogeneous. The accuracy of the results is verified by checking the unknowns with those available in the literature. The influence of the geometrical parameters on the displacements and the stress resultants are investigated by sensitivity analysis (Altekin and Yükseler, 2012).

In this study, fixed and simply supported annular plates carrying uniformly distributed load were investigated by approximate and numerical methods. The aim of this study is to examine the behavior of the annular plate under the large deflections. The numerical solution of the problem based on finite element method is obtained by using ANSYS and SAP 2000 software package. As the first step, the behavior of the plate under small deflections was studied. Then the behavior of the plate under large deflections was investigated. And maximum displacement values are calculated in the plate. Remarkable results are obtained in terms of structural behavior of the annular plates.

2 Bending Behavior of Circular and Annular Plates

When an applied loading and end restraints of the circular and annular plate are independent of the angle, the deflection of the plate will only depend upon the radial position. Plate Equation appears in the form;

$$\frac{1}{r}\frac{d}{dr}\left\{ r\frac{d}{dr}\left[\frac{d}{dr}(r\frac{dw}{dr}) \right] \right\} = \frac{q(r)}{D} \tag{1}$$

Solution of the homogeneous differential equation;

$$w = w_h + w_p \tag{2}$$

Homogeneous solution;

$$w_h = C_1 \ln r + C_2 r^2 \ln r + C_3 r^2 + C_4 \tag{3}$$

Where C_1, C_2, C_3 and C_4 are constants that can be evaluated from the boundary conditions.

The particular solution, the plate is under a uniform loading q(r) = q in this study;

$$w_p = \frac{qr^4}{64D} \tag{4}$$

Flexural stiffness (D);

$$D = \frac{Eh^3}{12(1-\upsilon^2)} \tag{5}$$

E= Young modulus, h= thickness of plate, υ= poisson ratio (this study υ=0.3)

2.1 Circular Plates

Circular plate of radius a under a uniform load q with clamped edge. In this case the terms involving the (ln) in Eqn. (3) yield an infinite displacement for all values of C_1 and C_2 so $C_1=C_2=0$.

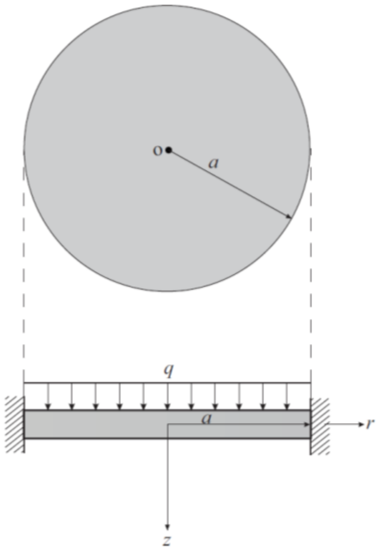

Figure 1. Circular Plate with clamped edge under a uniform load.

Boundary conditions are

$$w = 0\big|_{r=a}, \qquad \frac{dw}{dr} = 0\big|_{r=a} \tag{6}$$

The deflection is (small deflections case);

$$w = \frac{q}{64D}(a^2 - r^2)^2 \tag{7}$$

The maximum deflection is at the center of the plate (r=0)

$$w_{max} = \frac{qa^4}{64D} \tag{8}$$

Approximate formulas for uniformly loaded circular plates (clamped edge) with large deflections. Assuming that the shape of the deflected surface can be represented by the same equation as in the case of small deflections (Timoshenko, 1989)

$$w = w_0\left(1 - \frac{a^2}{r^2}\right)^2 \tag{9}$$

When large deflections are considered, obtaining a meaningful result may be impossible in many situations. Numerical calculation methods are used to solve this equation (Eqn.9) and equation is;

$$w_0 = \frac{qa^4}{64D}\frac{1}{1 + 0.488\frac{w_0^2}{h^2}} \tag{10}$$

2.2 Annular Plates

The bending of circular plates with concentric circular holes, this is called annular plates. Annular plate of inner radius b and outer radius a under uniform load q with outer edge is clamped and inner edge is free.

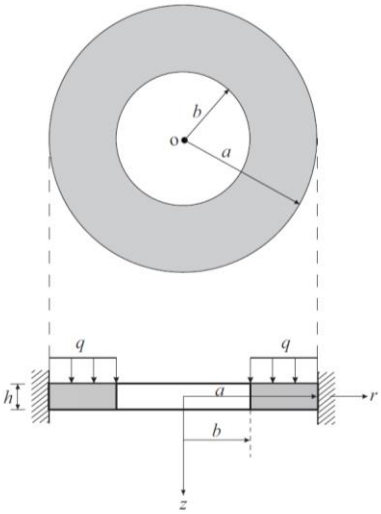

Figure 2. Annular Plate with clamped outer edge and free inner edge under a uniform load.

Boundary conditions are

$$w = 0\big|_{r=a}, \qquad \frac{dw}{dr} = 0\big|_{r=a} \tag{11}$$

$$M_r = 0\big|_{r=b}, \qquad Q_r = 0\big|_{r=b} \tag{12}$$

The second equation (Eqn.2) is solved using boundary conditions. The maximum deflection is at the inner edge of the plate (r=b) is determined (Timoshenko and Krieger, 1959).

$$w_{max} = k_1 \frac{qa^4}{Eh^3} \tag{13}$$

Note: k_1=0.0575 for a/b=2 and υ=0.3.

When large deflections are considered, calculations are not easy like circular plate. Numerical calculation methods are used to solve. The deflection of the plate is obtained by applying the principle of virtual displacements.

$$\frac{d(V + V_1)}{dw_0} \delta w_0 = 2\pi \int_b^a q\, \delta w\, r\, dr \tag{14}$$

The bending of circular plates with concentric circular holes, this is called annular plates. Annular plate of inner radius b and outer radius a under uniform load q with outer edge is simply supported and inner edge is free.

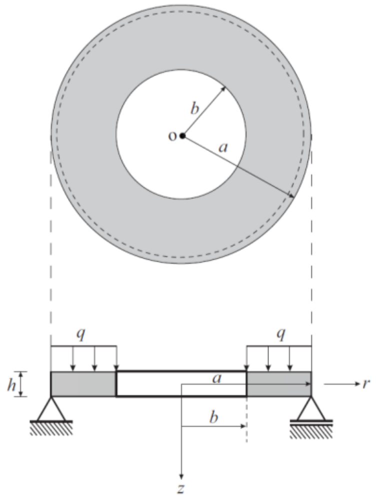

Figure 3. Annular Plate with simply supported outer edge and free inner edge under a uniform load.

Boundary conditions are

$$w = 0\big|_{r=a}\,, \qquad M_r = 0\big|_{r=a} \tag{15}$$

$$M_r = 0\big|_{r=b}\,, \qquad Q_r = 0\big|_{r=b} \tag{16}$$

The second equation (Eqn.2) is solved using boundary conditions. The maximum deflection is at the inner edge of the plate (r=b) is determined (Timoshenko and Krieger, 1959).

$$w_{max} = k_1 \frac{qa^4}{Eh^3} \tag{17}$$

Note: k_1=0.664 for a/b=2 and υ=0.3.

When large deflections are considered, calculations are not easy like circular plate. Numerical calculation methods are used to solve. The deflection of the plate is obtained by applying the principle of virtual displacements. (Eqn.14)

3 Finite Element Model

The modeling and analysis of annular plate have been performed based on ANSYS finite element software. SHELL 208 finite element in the ANSYS software is used for the finite element analysis. This element is suitable for modeling thin to moderately thick axisymmetric shell structures. This element has 2 node points and each node point has translations in the x, and y directions, and rotation about the z-axis. The structure of the SHELL 208 element is shown in Figure 4.

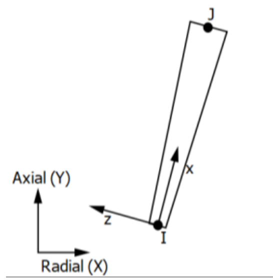

Figure 4. SHELL 208 element.

The model generated as a line structure. The line structure with boundary conditions is shown in Figure 5. Modeling structures such as annular plates can be generated using only one line because of the axisymmetrical properties. For this purpose, a symmetrical boundary condition must be assign at the inner edge of the annular plate.

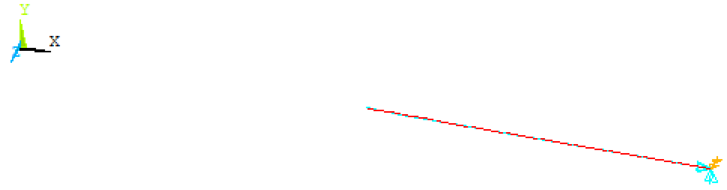

Figure 5. Modeling of annular plate as a line.

The completed annular plate geometry is shown in Figure 6. The optimum mesh distribution is chosen according to analyze results. Thus, using of 10 elements is sufficient on a line directed to the center of the annular plate.

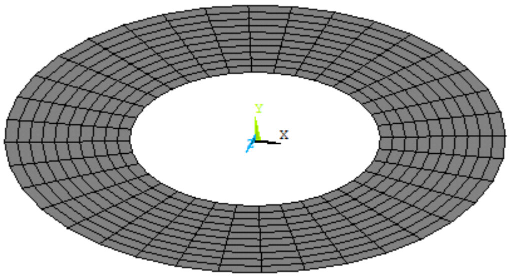

Figure 6. Annular plate geometry and mesh.

4 Numerical Results

The problem of circular plate in Figure 1 was solved, and the analytical results and the numerical results were compared. Then, the chosen plate is annular plate which is shown in the Figure 2 and 3. The material is homogeneous and isotropic. The parameters are as follows; E = 200000 N/mm^2, υ= 0.3, b= 500 mm, a= 1000 mm, h= 10 mm and q= 0.6 N/mm^2. The distribution of displacements for annular plates are shown in Figure 7.

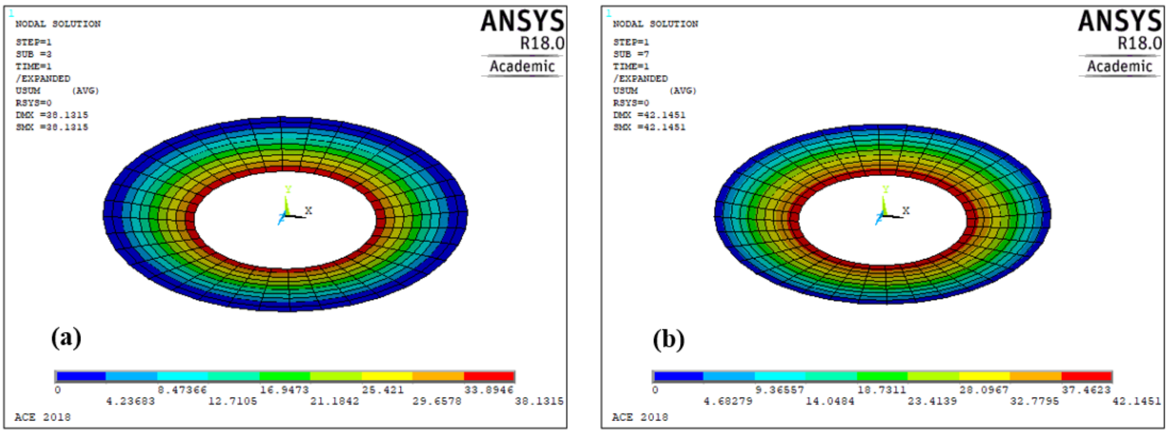

Figure 7. Distribution of displacements on the annular plates: a) annular plate with clamped at the outer edge, b) annular plate with simply supported at the outer edge.

In the verification problem, the maximum vertical displacement values for circular plates based on small displacement and large displacement plate theories are shown in Table 1. These values are measured at the center point of the circular plate.

Table 1. Deflection (w) at the center of the circular plate.

	Small deflection theory [mm]	**Large deflection theory [mm]**
Timoshenko and Krieger	511.875	45.713
ANSYS	512.125	42.3429
SAP2000	524.465	43.879

Vertical displacement values at the inner edge of the annular plate with clamped at the outer edge are shown in Table 2. These values are obtained with using small deflection and large deflection analyses for annular plates.

Table 2. Deflection (w) at the inner edge of the annular plate with clamped at the outer edge.

	Small deflection theory [mm]	**Large deflection theory [mm]**
ANSYS	172.703	38.1315
SAP2000	179.603	37.563

Vertical displacement values at the inner edge of the annular plate with simply supported at the outer edge are shown in Table 3. These values are obtained with using small deflection and large deflection analyses for annular plates.

Table 3. Deflection (w) at the inner edge of the annular plate with simply supported at the outer edge.

	Small deflection theory [mm]	**Large deflection theory [mm]**
ANSYS	2045.68	42.1451
SAP2000	2026.586	40.160

5 Conclusion

Numerical modeling and analysis of thin annular plates are investigated in this study. A numerical model is generated to understand the structural behavior of annular plates with using ANSYS software. The annular plates with distinct boundary conditions which have analytically impossible solutions can be studied with using this finite element model. Annular plates affected to distributed load and point load are analyzed with clamped and simply supported edge conditions. These results can be used as a verification of studies for annular plates. Large deflection analysis on annular plates generated dramatically realistic results. It is seen that, when the small displacement theory is preferred to design of structures, more resistant structures will be obtained. But, large deflection theory should be considered to design more economical engineering structures.

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